From: The Quantum Universe, by Tony Hey and Patrick Walters, Cambridge University Press, Cambridge, UK, (1987).

5

Quantum tunnelling

It is possible in quantum mechanics to sneak quickly across a region which is illegal energetically.

Richard Feynman

Barrier penetration

One of the most startling consequences of de Broglie's wave hypothesis and Schrödinger's equation was the discovery that quantum objects can 'tunnel' through potential energy barriers that classical particles are forbidden to penetrate. To gain some idea of what we mean by an energy barrier, let us go back to our roller coaster and look at a larger section of track, as shown in fig. 5.1. If we start the carriage from rest, high up on the left, at *A*, and ignore any small frictional energy losses, we know from the conservation of energy that we shall arrive on the other side at the same height we started from, at *C*. As we went over the little hill *B*, at the bottom of the valley, the car slowed down as some of our kinetic energy was changed to potential energy in climbing the hill, but because we started much higher up, we had plenty of energy to spare to get us over the top. However, if we started the carriage from rest at *A*, we do not have enough energy to climb over the hill *D* and get to *E*. This is an example of an 'energy barrier', and we can say that the region from *C* to *E* is 'classically forbidden'.

What is remarkable about quantum 'particles' is that they do not behave like these classical objects. An electron travelling on an 'electron roller coaster' of the same form as the roller coaster of fig. 5.1 can 'tunnel through' the forbidden region and appear on the other side! This 'barrier penetration' or 'quantum tunnelling' is now a commonplace quantum phenomenon. It forms the basis for a number of modern electronic devices such as the tunnel diode and the Josephson junction, of which more later. How can we obtain some understanding of how such tunnelling comes about, without presenting a detailed solution of the Schrödinger equation? One way of thinking about it uses an argument based on Heisenberg's uncertainty principle. In chapter 2 we phrased this in terms of the uncertainties in position and momentum measurements. However, another equivalent relation exists between uncertainties in measurements of time and energy

$$(\Delta E)\ (\Delta t) \approx h$$

Thus, although classically we can never change the total amount of energy

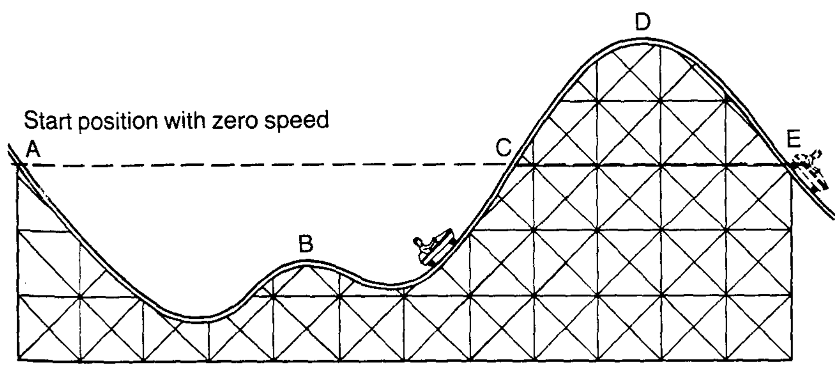


Fig. 5.1 An illustration of what quantum tunnelling means for a real roller coaster. If the carriage starts from rest at position A, *conservation of energy does not allow it to go higher than position* C *on the other side of the valley. In quantum theory, on the other hand, there is a chance that the carriage could 'tunnel through' the forbidden region between* C *and* E, *and emerge on the other side of the hill. For a real roller coaster this tunnelling is extremely unlikely!*

Fig. 5.2 In Mr Tompkins' wonderland, where Planck's constant is much larger, his car could tunnel through the wall, 'just like a good old ghost of the middle ages'.

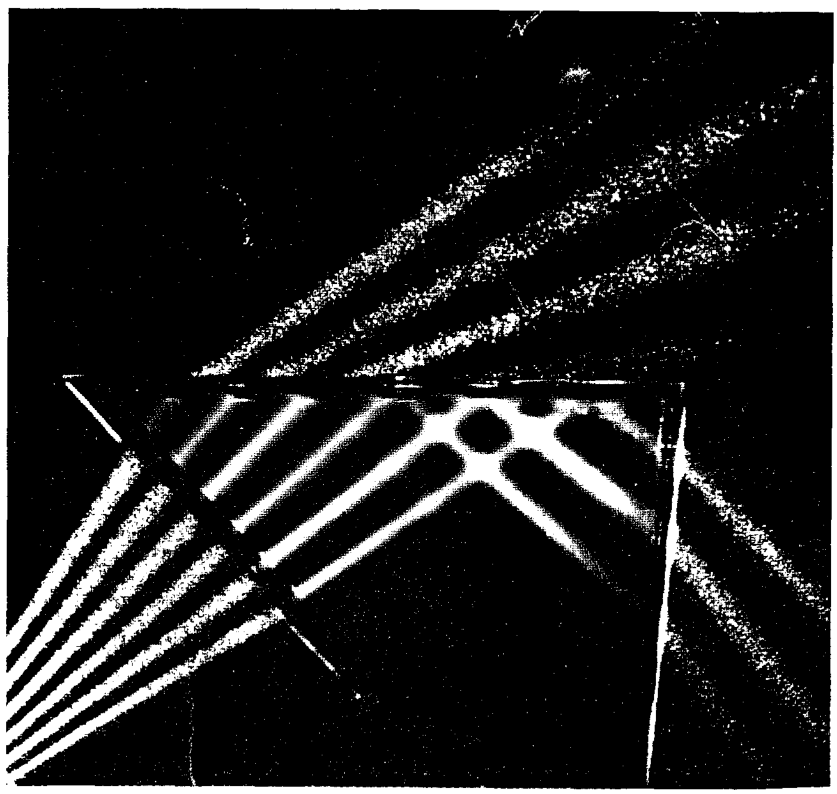
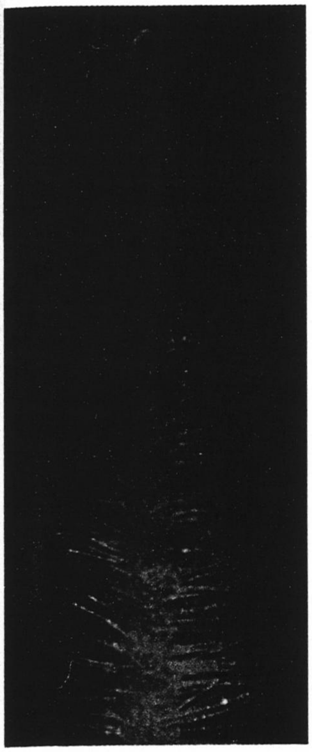


Fig. 5.3 This photograph shows several rays of light striking a prism at a variety of angles. As can be seen, beyond a certain 'critical' angle the light rays are entirely reflected and no light is transmitted through the prism.



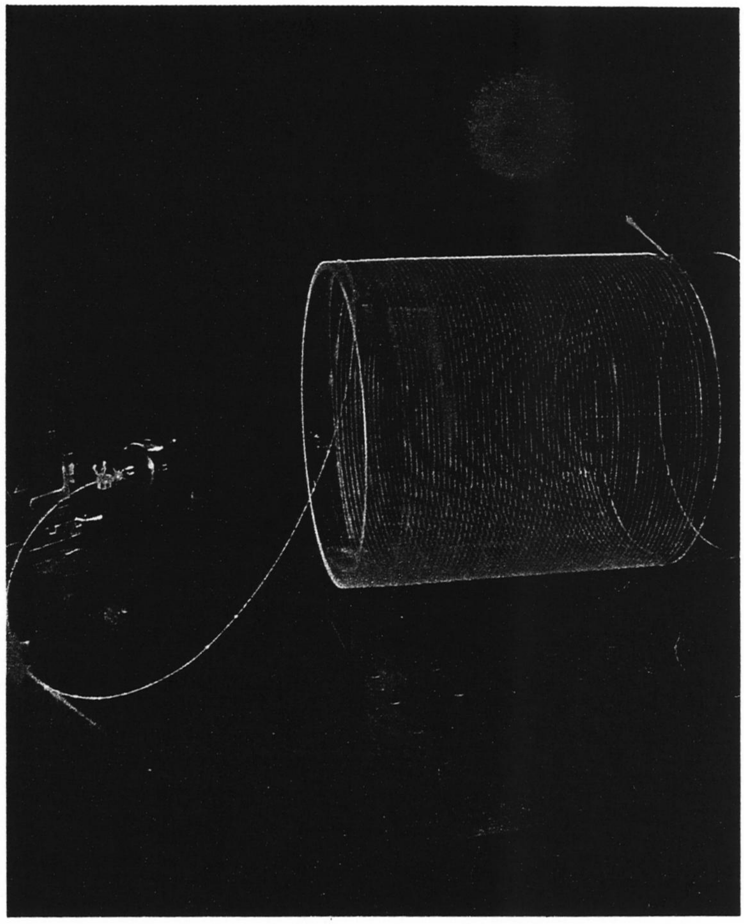


Fig. 5.4 The photograph on the right shows an optical fibre wound round a drum carrying light from a helium–neon laser. The fibre is about 100 m long and has been deliberately badly made so that some light leaks out of the sides so that we see the fibre as red. In a high quality fibre nearly all the light would emerge from the end. In this case, the light coming from the end of the fibre is directed onto a screen. The photograph above shows red laser light guided through the tissue of a corn root seedling. Tiny root 'hairs' pipe the light out to the tips of the hairs.

without violating the conservation of energy, in quantum mechanics, if the time uncertainty is Δt, we cannot know the energy to better than an uncertainty $\Delta E = h/\Delta t$. Roughly speaking then, we can 'borrow' an energy ΔE to get over the barrier so long as we repay it within a time $\Delta t = h/\Delta E$. If the barrier is too high or too wide, therefore, tunnelling becomes extremely unlikely and all the electrons will be reflected, just like the roller coaster car. Needless to say, this sort of 'hand-waving' argument must be backed up by detailed calculation with the Schrödinger equation, but such arguments do give us some sort of insight into quantum tunnelling. It is perhaps more illuminating, however, if we look at the behaviour of more familiar waves. This phenomenon of tunnelling is then seen to be a general property of wave motion – it only becomes surprising when taken in conjunction with de Broglie's hypothesis that all quantum 'particles' have wavelike properties.

Wave tunnelling

Although both waves on a string and water waves can be made to exhibit 'wave tunnelling', probably the most familiar example involves light in its wavelike guise. Consider what happens when light travels from air into a block of glass. As shown in fig. 5.3, because light travels slower in glass than in air, the wave slews round and the light changes direction. This is called 'refraction'. Now consider light travelling from glass to air. Instead of being bent towards the vertical, the light is bent away from it. If we increase the angle at which we shine the light on the glass–air surface, there will be an angle – the 'critical' angle – at which the light emerges in the air just grazing

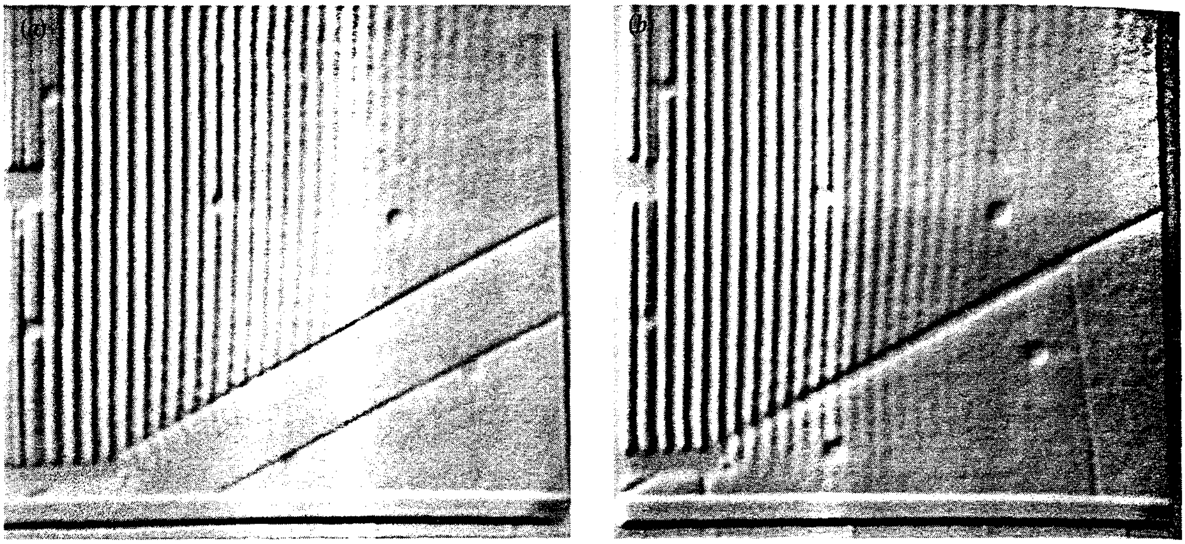
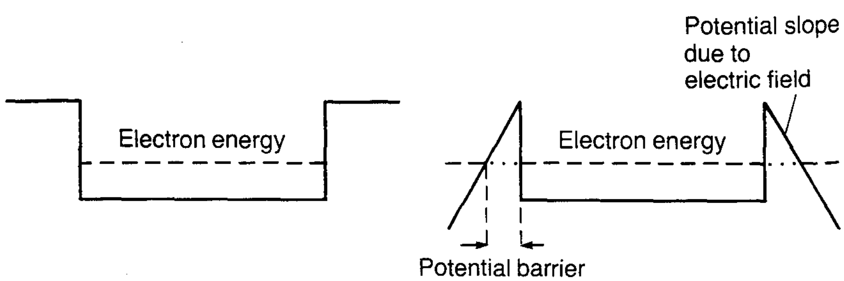


Fig. 5.5 Tunnelling with water waves. (a) The speed of these water waves depends on the depth of water. This photograph shows the water waves undergoing 'total internal reflection' at a barrier consisting of a change in water depth. Notice that there is some kind of disturbance in the forbidden region beyond the barrier but that it does not correspond to an ordinary wave. (b) This photograph shows that as the width of the forbidden region is decreased, the water wave can 'jump the gap' and appear on the other side. This standard wave phenomenon is the basis for tunnelling in quantum mechanics.

the surface. What happens if we increase the angle still further? What must happen is that all the light is reflected from the glass–air surface and no light escapes into the air. This is called 'total internal reflection' and is the basis for modern fibre optics. How is all this connected with quantum tunnelling? Well, although no light rays penetrate the air beyond the glass when the light arrives at an angle larger than the critical angle, there is nonetheless some sort of wave disturbance in the air. This is not a wave that carries energy, like ordinary 'travelling' waves, but a sort of 'standing' wave pattern that does not transmit any light energy. The wave patterns on a string fixed at both ends are examples of standing waves. However, the type of standing wave involved here – a so-called 'evanescent' wave – is special in that the disturbance dies away very rapidly the further away we go from the surface. The connection with tunnelling comes about if we bring up another block of glass parallel to the first one. As we bring the two blocks towards each other,

Fig. 5.6 The diagram on the left shows a simplified picture of the potential well for electrons in a metal. The broken line represents the electron energy which is insufficient to escape from the well. On the right, however, we show how the potential is modified in the presence of a large electric field. Electrons can now escape from the metal by tunnelling through the barrier.



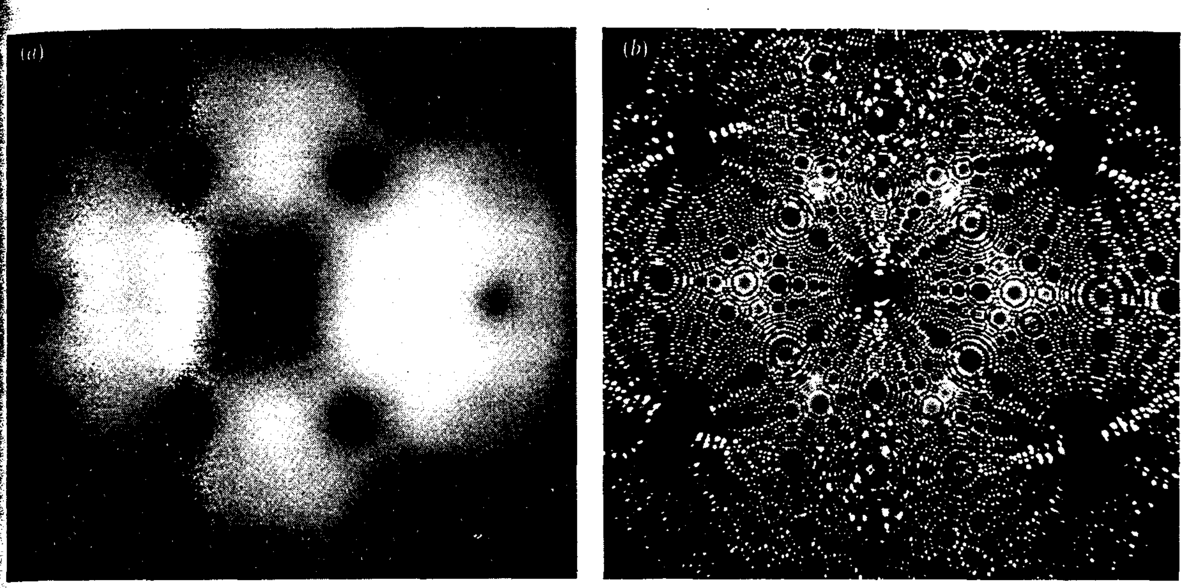


Fig. 5.7 (a) *The left hand picture is the tip of a tungsten needle photographed with an electron field-emission microscope. The needle is negatively charged so that there is a very large electric field at the tip. Electrons tunnel out and are accelerated away from the needle by the high field. This produces a very high magnification of the needle showing where the electrons are most densely clustered. The resolution is not quite high enough to see individual atoms: the bright areas in the photograph correspond to electron emission from the atoms at the corners and edges of the layers making up the tip of the needle.* (b) *The right hand picture is a helium ion emission microscope photograph of the tip of a tungsten needle. In this case, the needle is positively charged and surrounded by helium gas. Electrons are captured by the high fields at the tip of the needle and the remaining positively charged helium ions accelerated away. The helium ions are heavier than the electrons used above and suffer less from random sideways motion as well as having a smaller quantum wavelength. These two improvements combine to improve the resolution sufficiently so that individual atoms may be seen. Each bright spot on the picture corresponds to a tungsten atom. The pattern of rings in the image can be understood by imagining the atoms in a metal to be packed in regular layers, like apples in a box. If we now imagine cutting a cone shape from the metal to form the tip of a needle, there will be circular rings as each layer is exposed.*

the evanescent wave begins to penetrate the second block and a transmitted ray of light appears! The closer the two blocks are brought together, the more light energy that reappears as a transmitted ray. This is because the amplitude of the standing wave in the 'forbidden' air gap has not had time to decay away so much. Physicists call this phenomenon 'frustrated total internal reflection' and it is an exact analogue of quantum barrier penetration or tunnelling for de Broglie waves. Fig. 5.5 shows a ripple tank photograph of barrier penetration with water waves. This phenomenon is also useful in modern optics as a 'beam splitter': the amount of light transmitted can be controlled by adjusting the width of the forbidden gap.

Applications of quantum tunnelling

There are many devices now in common use that rely on the ability of quantum particles to tunnel through barriers. The two examples we shall describe here both involve electrons, but there are other examples that we shall come across later in which alpha particles and pairs of electrons are doing the tunnelling. Our first example concerns the so-called 'field-emission' microscope. In a metal, the electrons that carry the electric currents are able to move about relatively freely. As a very simplified model of a metal, the electrons can be imagined as moving about in an attractive potential well, rather like the quantum 'box potential' we talked about in the previous chapter. Since it takes energy to knock electrons out of the metal, there must be an electrical 'hill' or barrier at the sides that keeps them in. If we now turn on a strong electric field, there is a large attractive force that wants to drag electrons out of the metal. The resulting potential looks like fig. 5.6*b*. As can be seen, there is now a barrier that the electrons inside the metal can tunnel through and escape. Using the Schrödinger equation we can calculate the probability for this to happen. One finds that a very large electric field is needed for there to be appreciable tunnelling. Such large fields can be produced at the tip of a sharp needle of metal, and experiments in 1928 confirmed the predictions of quantum mechanics for this 'field-emission' process. Several years later, this effect was used as the basis for a new form of microscope. After tunnelling through the barrier, the electrons will move away from the needle in a straight line. Thus, if we surround the needle by a phosphorescent screen we will see a greatly enlarged image of the tip of the needle caused by the electrons arriving at the screen. Electrons are most easily emitted from the corners and edges of layers of atoms in the tip, and these positions show up brightest in the image. Magnifications of up to one million can be reached in this way.

An even more sensitive device was later developed from this field-emission microscope. Instead of electrons arriving at the screen, the 'field-ion' microscope uses helium ions. This comes about as follows. Instead of a vacuum surrounding the needle, a small amount of helium gas is allowed in and, in addition, the field on the needle is reversed. When a helium atom collides with the tip, the high electric field not only ionizes the helium but accelerates the positively charged ion away from the needle. Because the helium ions are much more massive than electrons (about 8000 times heavier) they have much larger momentum and a correspondingly shorter de Broglie wavelength. Thus, this 'field-ion' microscope is able to pick out much finer details of the structure of the tip of the needle. The bright spots on the image reveal the positions of individual metal atoms on the surface.

A new type of microscope that relies on quantum tunnelling has recently been developed. This is the so-called 'scanning tunnelling microscope', which enables one to achieve magnifications of up to 100 million and to reconstruct the surfaces of solids atom by atom. The basic idea is very simple. According to quantum mechanics, electrons in a solid have a small but non-vanishing chance of being found just outside the metal surface. The probability for this to happen falls off very rapidly with distance away from the surface. Now, if a needle-like probe is brought up very close to the surface, and an electrical voltage applied between it and the metal, a tunnelling current will flow across the gap between them. Since the magnitude of this current is extremely sensitive to the distance of the probe from the metal, it is possible to scan across the metal systematically and to reconstruct a very accurate contour map of the surface. This type of microscope is much more versatile than the field-ion microscope and promises to have many new applications in physics, chemistry and biology.

Our second example of barrier penetration by electrons is a device used in modern electronics called the 'tunnel diode'. This consists of a junction between two types of semiconductor to which a voltage can be applied. We shall explain how semiconductors work in the next chapter – here all we need to know is that the electrons carrying the current 'see' a potential barrier like a wall. Electrons can tunnel through the barrier from one side of the junction to the other, but the amount of this 'tunnelling' current depends very sensitively on the height of the barrier. One can therefore make a very rapid 'switching' device in which the amount of current allowed to flow through the device is varied by changing the applied voltage and thus altering the height of the barrier.

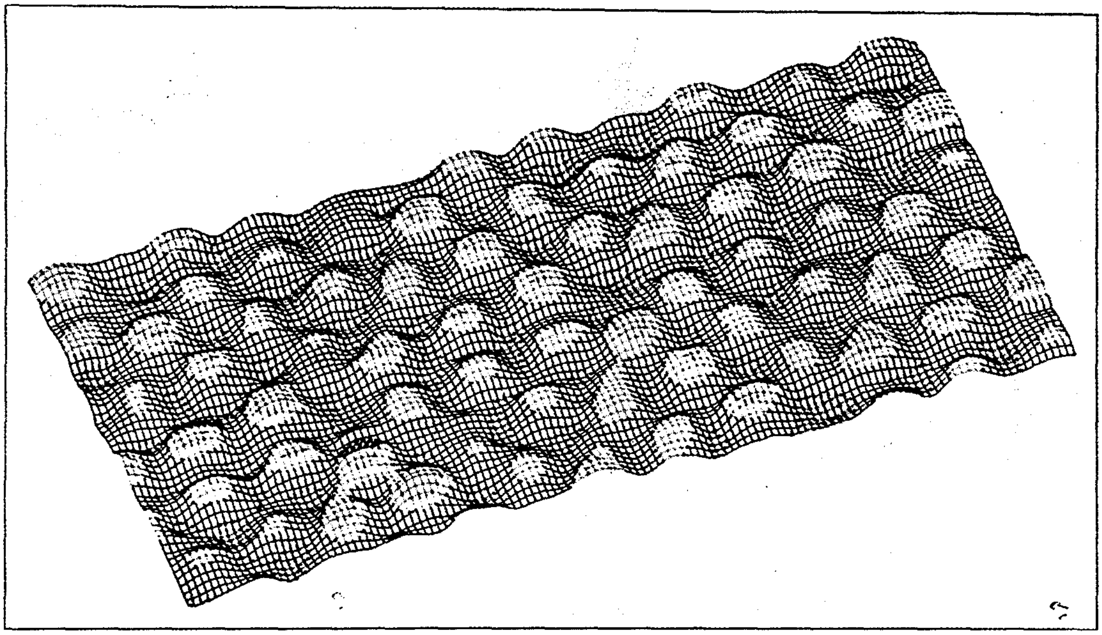


Fig. 5.8 A scanning tunnelling microscope consists of a very sharp needle point that surveys the surface of a specimen very accurately. When a high voltage is applied between the surface and the needle, electrons can tunnel from the tip of the probe to the specimen under investigation. This tunnelling current is very sensitive to the height of the probe above the surface. In the microscope, the height of the needle can be adjusted as the needle moves over the surface so that the current remains constant. In this way, the up and down movements of the needle map out the detailed contours of the surface. The colour coded picture shows the surface of silicon with each of the yellow bumps representing a silicon atom. A regular diamond structure consisting of 12 atoms of silicon may be picked out.

Nuclear physics and alpha decay

One of the great puzzles in the early days of nuclear physics concerned alpha decay. The puzzle was this. In the radioactive decay of uranium, physicists had measured the energy of the alpha particle that was thrown out of the nucleus and found it to be about 4 MeV. A quick word about units of energy is needed here. An electron-volt, or 'eV', is the amount of energy an electron gains in 'sliding down' a potential hill one volt high. This amount of energy is typical of the energies of electronic energy levels in atoms. For processes involving the nucleus, on the other hand, the energies are much larger, and a convenient unit of energy is a million electron-volts, or 'MeV' for short. Now back to the story. Rutherford had done experiments shooting alpha particles at atoms and had found that alpha particles with about 9 MeV of energy were repelled by the positive charge of the nucleus. In other words, to get inside the nucleus requires much more energy than the 4 MeV observed for alpha particles emitted in radioactive decay. To make this problem more