KIRCHHOFF'S LAWS

Lab Goals: Learn how to analyze more complicated circuits with more than one voltage source and numerous resistors.

Lab Notebooks: Write descriptions of all of your experiments in your lab notebook. Answer all of the questions about the lab in your notebook also, instead of in the lab writeup.

Apparatus Power supply, 1.5V battery, breadboard, resistors, hookup wires, multimeter.

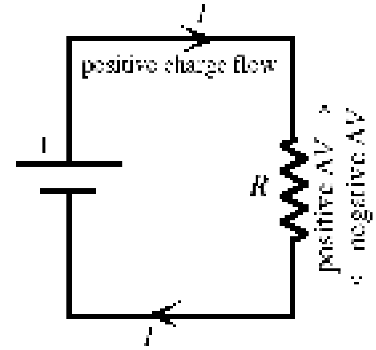


Figure 1

Introduction By now you're well acquainted with the governing relationship for a resistor, $I = \Delta V / R$, known as "Ohm's Law". But as we use it in today's lab we'll need to be careful about sign. When we speak of current I, we mean **conventional current**, the charge flow assuming that it is the *positive* charges free to flow in our conductors. If the charges actually free to move are negative, then if a battery were connected, these negative charges would move toward the positive side of the battery. If the free charges are positive, they would move toward the battery's negative. But *nothing we do in our lab exercise would be different either way*, and so we adopt the less error-prone route of assuming positive charge flow. Therefore, when a battery is connected to a conductor, a current I of positive charges flows from the battery's plus toward its minus.

Having settled on conventional current, we now recall that charges lose energy to heat as they move through a resistance, and accordingly the potential drops as they move. Therefore, we may write Ohm's Law in a way more useful in circuit analysis. As we move in the direction of the current through a resistor, there is a drop in potential governed by:

$$\Delta V_{\substack{drop \text{ in the} \\ \text{direction of } I}} = I_{\text{through R}}\, R \qquad (1)$$

Today's lab has two goals: To see how multiple resistors behave as a single resistor, and to understand how to determine current flow in more complicated circuits that cannot be analyzed by treating multiple resistors as one. The only thing we really need to add to equation (1) is a principle we've already encountered: Charge cannot perpetually build up anywhere in a circuit, so:

$$I \text{ going into any point in a circuit} = I \text{ going out} \qquad (2)$$

Obviously the circuit of Figure 1 obeys this principle, for in a simple series circuit, the current must be the same everywhere . We'll soon see how we use the principle in more complex circuits.

You've already dealt with the ideas of series and parallel in several places in lab. Still, let's formalize the ideas with respect to finding equivalent resistance.

Resistors in Series

Figure 2 shows two resistors in series, for there is no place for current to split; the same current I must flow through one and then the other. It flows from top to bottom through the resistors, so there is a potential *increase* from the bottom to the top of the bottom resistor, and a similar increase across the top resistor. These two must add up to the potential difference established by the battery. Combining this with the fact that each resistor is independently governed by equation (1) gives

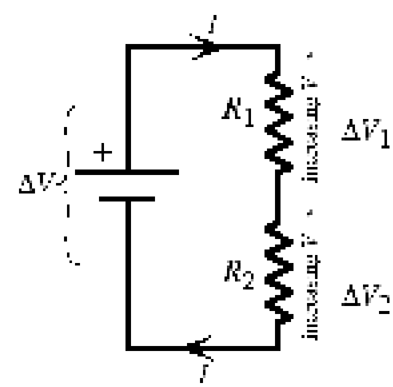


Figure 2

$$\Delta V = \Delta V_1 + \Delta V_2 = I\,R_1 + I\,R_2 = I\,(R_1+R_2) \qquad \text{or} \qquad \frac{\Delta V}{I} = R_1 + R_2$$

We see that the battery's ΔV causes a current I exactly what it would be if connected to a single resistor whose resistance is the sum of the two. In general,

$$\text{In Series:} \qquad R_{\text{effective}} = \sum_i R_i \qquad (3)$$

Resistors in Parallel

Figure 3 shows two resistors in parallel, for current may split at the point labeled *a*, going through one resistor or through the other, before reuniting at point *b* to return to the battery. In a series connection each resistor has the same current. But in parallel it is the potential difference that is the same for both, while the two currents, according to principle (2), add to give the current I flowing from the battery. Again applying equation (1) to each resistor,

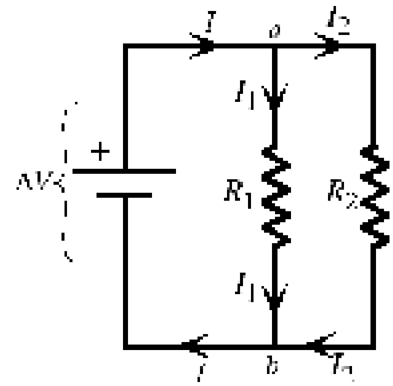


Figure 3

$$I = I_1 + I_2 = (\Delta V_1/R_1) + (\Delta V_2/R_2) = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{or}$$

$$\frac{\Delta V}{I} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

So far as the battery is concerned, its potential difference ΔV causes a current I exactly what it would be if connected to a single resistor whose resistance is the reciprocal of the sum of the reciprocals. In general:

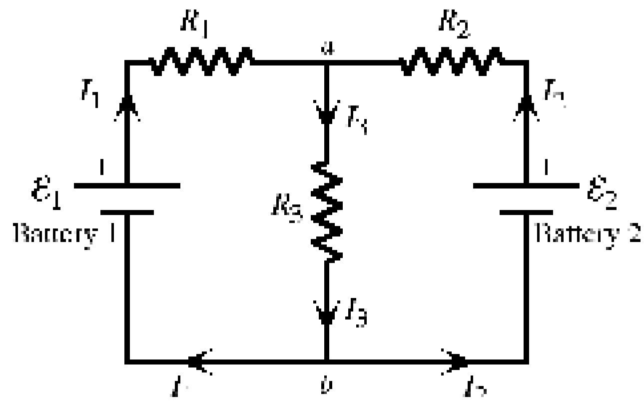


Figure 4

$$\text{In Parallel:} \quad R_{\text{effective}} = \frac{1}{\sum_i 1/R_i} \qquad (4)$$

Note that, when resistors are in parallel, the effective resistance is always smaller than *any* of the individual resistances. We should expect this, for we noted in the first lab that resistance in parallel makes for *easier* current flow. We also noted that the ratio in which the current splits follows from knowing that the resistors indeed have the same ΔV and that it equals I times R, so that the current ratio is the inverse of the resistance ratio.

A Method Most General

If it is just one battery connected to many resistors, usually you can break a circuit down into combinations of resistors in parallel and series to find the current flowing from the battery. But if there are multiple batteries, this is often impossible. The simplest example, which you will do in this lab, is shown in Figure 4. No two resistors are in series, for no two must, as in Figure 2, have the same current flowing through them. Rather, current has places where it can split between them. And no two are in parallel, for no two necessarily have the same potential difference across them, as in Figure 3, where the resistors' tops are connected by a resistanceless wire, ensuring equal potential, and their bottoms are similarly at equal potential.

In cases like this, we fall back on a more general, if more tedious, method: adding potential differences around loops. Potential is a well-defined thing. Provided some place is chosen as zero potential, the potential is set everywhere else in the universe. If we start somewhere, and keep account of how the potential increases and decreases as we move about randomly, then if we eventually return to where we started, the increases and decreases had better add to zero, for the potential is back where it started. How do we apply this to a circuit?

1. Choose appropriate symbols and directions for the yet-unknown currents. In Figure 4 they've been chosen arbitrarily. Because current can split at points *a* and *b*, we must allow for different currents in different parts of the circuit. But points *a* and *b* are the only "branching points", so things elsewhere are pretty much in series. For instance, whatever current leaves Battery 2 must flow through resistor R_2—no more, no less—and it must also be what leaves point *b* headed for Battery 2, for nowhere from point *b* to point *a* along the rightmost part of the circuit are there places where current can split. Similarly, the current must be the same everywhere from *b* to *a* along the leftmost part of the circuit. But how do we know the current is moving in the directions we've chosen? We don't—but we don't have to! The method always works, simply by choosing directions arbitrarily and following through. In the end we discover the true directions very easily: If the numerical values turn out negative, it means the direction is opposite what was arbitrarily chosen. Still, though arbitrarily chosen, the currents must "add up". Principle (2) must be obeyed. At point *a*, I_1 and I_2 enter, and I_3 leaves. Conversely, at point *b*, I_3 enters and I_1 and I_2 leave. At either point, the Principle requires that:

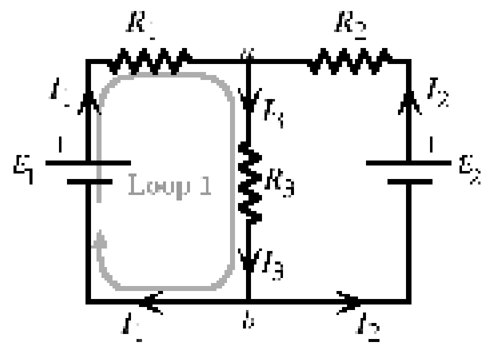


Figure 5

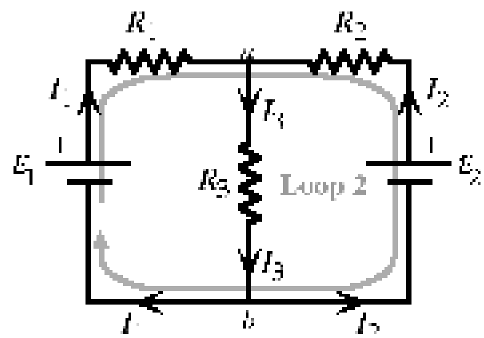


Figure 6

$$I_1 + I_2 = I_3 \qquad (5)$$

2. Add up potentials around "loops", however many loops are needed to include at least some information about every part of the circuit. Figure 5 shows a clockwise loop, starting at the negative of Battery 1, going through the center part of the circuit, then back where it started. Here is the accounting of potential increases and decreases:

Loop 1: $+\mathcal{E}_1 - I_1 R_1 - I_3 R_3 = 0 \qquad (6)$

From the negative to the positive of Battery 1, the potential goes up by $\mathcal{E}_1$. (This new symbol stands for the battery's "emf", meaning **e**lectro**m**otive **f**orce, the magnitude of the potential difference it establishes. Naturally it's measured in volts.) Moving in the direction of I_1, the potential *drops* across resistor R_1 by the appropriate current times resistance—this is equation (1)! Moving in the direction of I_3, the potential similarly drops across R_3, and then the potential changes no more in returning to the negative of Battery 1, for potential never changes along resistanceless wires. Ending where we started, the total change in potential must be zero.

But not all the circuit has been considered yet—what of R_2? One more loop is enough, so long as it covers that part of the circuit. Figure 6 shows one, and the accounting is as follows:

Loop 2: $+\mathcal{E}_1 - I_1R_1 + I_2R_2 - \mathcal{E}_2 = 0$ (7)

The first two terms are just as before, for this loop starts out the same way, but it goes across resistor R_2 *opposite* the current, so it goes toward the higher potential side, thus the plus sign in the third term. The fourth and last term is negative because the loop takes us from the high side to the low side of this battery. It makes no difference what's going on elsewhere in the circuit or which way currents flow—a battery has a plus side and a minus side, which never vary!

Conclusion And what good is all this? Well, assuming that the causes of current flow, $\mathcal{E}_1$ and $\mathcal{E}_2$, are known, as are R_1, R_2 and R_3, equations (5), (6) and (7) are three equations in three unknowns, I_1, I_2, and I_3. We can solve for the effects, i.e., the currents. When solved, they yield:

$$I_1 = [\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2R_3]/Z, \quad (8a) \qquad I_2 = [\mathcal{E}_2(R_1 + R_3) - \mathcal{E}_1R_3]/Z, \quad (8b) \qquad I_3 = [\mathcal{E}_1R_2 + \mathcal{E}_2R_1]/Z \quad (8c)$$

$$\text{where } Z \equiv R_1R_2 + R_1R_3 + R_2R_3$$

LAB EXERCISE: KIRCHHOFF'S RULES

Introduction The analysis of the circuit in Figure 4 (the same as in Figure 8) is usually referred to as "applying Kirchhoff's Rules". In short, it involves writing expressions accounting for the fact that (1) the currents and (2) the potential differences must add up. From these Rules, the flow of current in even the most complex circuits can be predicted, and in this activity we verify them in a case where the alternative, a simple "series and parallel" approach, won't work. By now, you're expected to be able to wire a circuit from a standard circuit diagram and to use the multimeter to measure currents and potential differences without many reminders. But here's something from the "Simple Circuits" lab you may not remember but you'll certainly need to know. When measuring potential difference, a positive value means that the VΩmA jack is at higher potential than the COM jack; when it reads a negative, VΩmA is at lower potential than COM. And when measuring current, a positive value means that current is flowing *into* the VΩmA jack (and out the COM jack); a negative means current flows *out* the VΩmA jack.

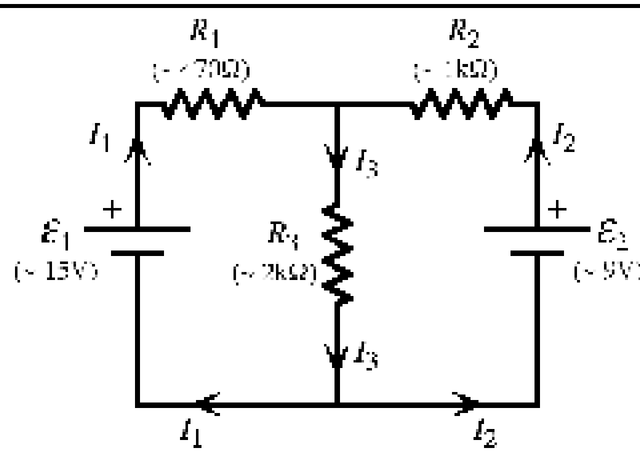


Figure 7

Important Note: This lab writeup often suggests that you measure the current directly using your multimeter. Since we now believe Ohm's law, it is much easier to measure voltage differences directly and then use Ohm's law to compute the currents.

Procedure 1. Wire together the circuit of Figure 8. $\mathcal{E}_1$ is the power supply, and $\mathcal{E}_2$ is the battery. R1=470Ω, R2=1000Ω, and R3=2000Ω. Check the resistor color codes, and also measure the resistor values with the multimeter before you put the resistors into the breadboard. With the multimeter dial set at 2000 in the Ω sector and its leads in VΩmA and COM jacks, measure R_1, R_2 and R_3. (For R_3 you may have to change to the 20k scale in the Ω sector.)

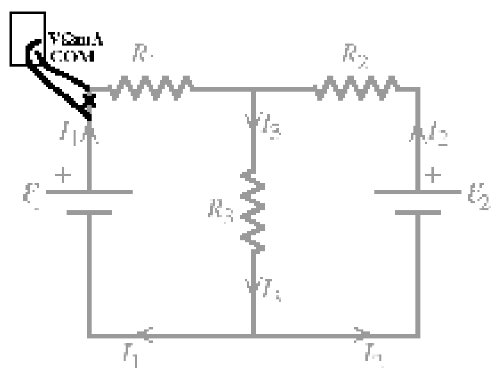
2. Set the power supply voltage to 15V and measure it with the multimeter.

3. You already know the values of the resistances. Now that you also know the batteries' emfs, what theoretically *should* be the currents (don't measure them yet!) I_1, I_2 and I_3, including sign? Don't *solve* equations (5), (6) and (7); *merely plug in* to their result: equations (8a), (8b), and (8c). Show your calculations in your lab notebook..

   Theoretical: I_1:_________mA I_2:_________mA I_3:_________mA

4. Now measure the currents, but as you do, keep track of sign/direction as follows. The figure below is one way of representing a measurement of I_1, showing the important things: where the circuit is being broken and which multimeter lead is where. Similarly *modify this figure as necessary* to represent all of your current measurements. Then in the space below record the values you read, *including sign exactly as you read it*,

Measured:
I_1:_________mA
I_2:_________mA
I_3:_________mA



5. Discuss how your measured currents compare with the theoretical values and what the signs mean. If you see significant discrepancies, how do you resolve them? [Again it should be helpful to your answer to calculate percent differences.]

6. Now that you know the currents, what should be (don't measure them yet!) the magnitudes of the potential differences across each of the three resistors and which side should be higher potential? Show your calculations.

Magnitude of potential difference	Higher potential side (circle one)	
ΔV_{R1}:____________V	Left	Right
ΔV_{R2}:____________V	Left	Right
ΔV_{R3}:____________V	Top	Bottom

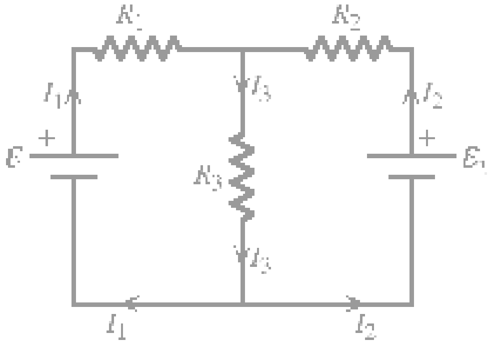
7. Now, as you did in step 4, modify the figure below to show how you measure potential differences across the three resistors, then record your values, including the sign that you read from the meter.

Measured:

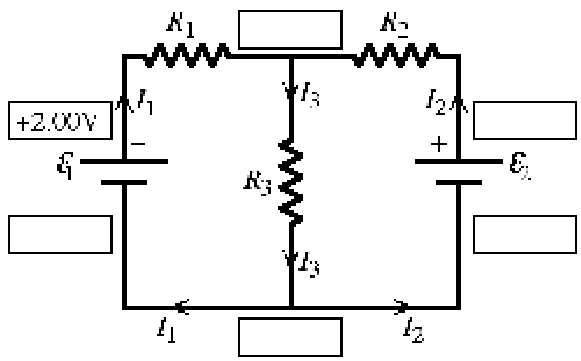
ΔV_{R1}:_________V

ΔV_{R2}:_________V

ΔV_{R3}:_________V



8. Discuss how your measured potential differences compare with the theoretical values and what the signs mean. If you see significant discrepancies, how do you resolve them?
9. Now, it's easy to lose sight of what's really behind circuit analysis. Let's make the ideas a bit more tangible. You have only potential *differences*. But suppose we arbitrarily *define* the potential V to be exactly +2.00V at the positive side of Battery 1. As we know, once done, this determines the potential *everywhere*. Using your *experimentally measured* potential differences, fill in the boxes, showing the actual value the potential would be at all the indicated points in the circuit. (Note: Does negative V bother you? We could define the *gravitational* potential energy to be zero at the top of Mt. Whitney, couldn't we?)



10. Within reasonable round-off/significant-figures error, does everything fit?

LAB EXERCISE: POWER

Procedure

1. In the previous lab exercise, Battery 1 takes positive charges at low energy at its negative terminal and shoots them out at high energy at its positive terminal. The charge per unit time I_1 times the energy difference per charge, $\mathcal{E}_1$ both of which you measured in Activity 2A, is an energy per unit time, a power. (Power = $I \times \Delta V$.) How much power is Battery 1 putting into the circuit? Power put into circuit by Battery 1:____________W
2. Resistors just turn electrical power into heat, and it should be possible to calculate the power dissipated in any of three equivalent ways,

   Power dissipated: $(I_{\text{through } R})(\Delta V_{\text{across } R}) = (\Delta V_{\text{across } R})^2/R = (I_{\text{through } R})^2 R$

   They're equivalent because equation (1) can be used to convert any one into any other. Using your *experimentally measured* values of current and potential difference for R_1, calculate the power dissipated each of these ways. It's a loss, but just report the absolute value.

   Power dissipated in R_1: __________W __________W __________W
3. Do they indeed agree within reasonable round-off/significant-figures error?
4. Using your favorite method, calculate the power dissipated in resistor R_2 and in resistor R_3.

   Power dissipated in R_2: __________W Power dissipated in R_3: __________W
5. The circuit is neither gaining nor losing energy—it's a "steady state". Do the four powers you've calculated add up, and if not, can you account for the discrepancy *quantitatively*?