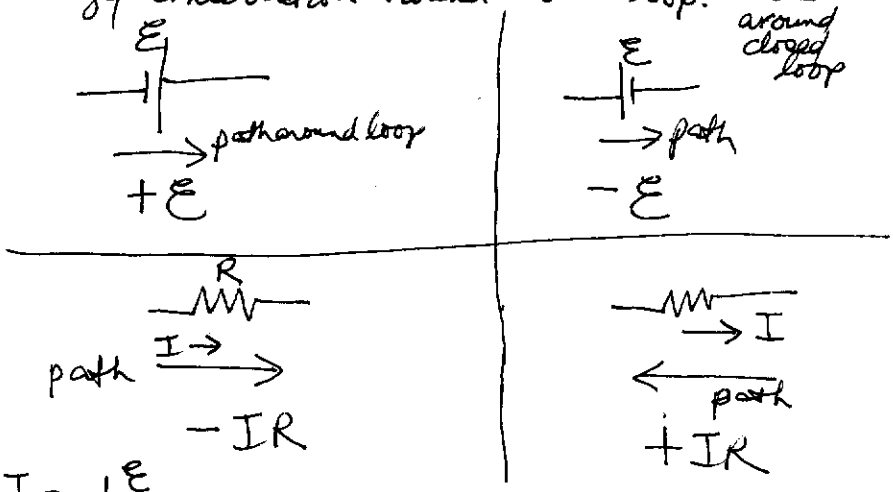
Kirchhoff's laws

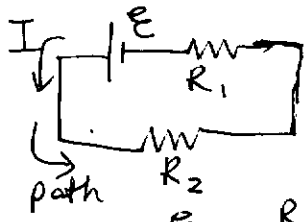
1) Charge conservation at node $\boxed{\sum I_{in} = 0}$ Sum of currents coming into a node $= 0$.

or $\boxed{\sum I_{in} = \sum I_{out}}$ Sum of currents coming into node = sum of current leaving node.

2) Energy conservation around closed loop. $\sum \Delta V = 0$. around closed loop

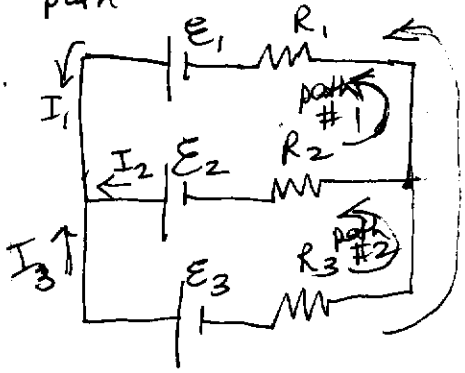


ex 1.



$\mathcal{E} - IR_1 - IR_2 = 0$, $I = \dfrac{\mathcal{E}}{R_1 + R_2}$

ex 2.



Find I_1, I_2, I_3 , 3 unknowns

① $I_1 + I_2 + I_3 = 0$

② $\mathcal{E}_1 - I_1R_1 - \mathcal{E}_2 + I_2R_2 = 0$ Path #1

③ $\mathcal{E}_2 - I_2R_2 - \mathcal{E}_3 + I_3R_3 = 0$ Path #2

④ $\mathcal{E}_1 - I_1R_1 - \mathcal{E}_3 + I_3R_3 = 0$ Path #3

Note eq. ④ = eq ② + eq ③ . So, 3 independent equations are, e.g. #1, #2, #3; or #1, #3, #4 ; or #1, #2, #4.

② $\Rightarrow$ $\mathcal{E}_1 - I_1R_1 = \mathcal{E}_2 - I_2R_2$ (2') $I_2 = \dfrac{\mathcal{E}_2 - \mathcal{E}_1 + I_1R_1}{R_2}$

③ $\Rightarrow$ $\mathcal{E}_2 - I_2R_2 = \mathcal{E}_3 - I_3R_3$ ②+③ $\Rightarrow$ $I_3 = \dfrac{\mathcal{E}_3 - \mathcal{E}_1 + I_1R_1}{R_3}$

$$I_1 + I_2 + I_3 = 0 = I_1 + \frac{\mathcal{E}_2 - \mathcal{E}_1 + I_1R_1}{R_2} + \frac{\mathcal{E}_3 - \mathcal{E}_1 + I_1R_1}{R_3} = 0$$

$$I_1R_2R_3 + \mathcal{E}_2R_3 - \mathcal{E}_1R_3 + I_1R_1R_3 + \mathcal{E}_2R_2 - \mathcal{E}_1R_2 + I_1R_1R_2 = 0$$

$$I_1 = \frac{\mathcal{E}_1(R_2+R_3) - \mathcal{E}_2R_3 - \mathcal{E}_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}, \quad I_2 = \frac{\mathcal{E}_2(R_1+R_3) - \mathcal{E}_1R_3 - \mathcal{E}_3R_1}{R_1R_2 + R_1R_3 + R_2R_3},$$

$$I_3 = \frac{\mathcal{E}_3(R_1+R_2) - \mathcal{E}_1R_2 - \mathcal{E}_2R_1}{R_1R_2 + R_1R_3 + R_2R_3}$$

Check $I_1 + I_2 + I_3 = 0$