

Bandpass Filter

Circuit 2: RLC resonant circuit

An inductor and capacitor connected either in series or parallel will resonate at a characteristic frequency:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

Derive this relationship, starting with the complex impedances for the components, for a parallel LC circuit. What does the impedance across the parallel LC circuit do as you approach the resonant frequency? What if there is some loss (consider a parallel high resistance R)?

Using the supplied 150μH ferrite core inductor and 20-400pF variable capacitor (C_t in figure below), construct a LC parallel circuit. What is the expected voltage across this parallel LC circuit vs frequency? Hint: this is related to the impedance vs frequency. In reality, your LC circuit is not completely free of loss. Estimate and list all sources of resistance. Next, excite the LC oscillator. Inductively couple a little oscillator power into this circuit via ~1-2 turns of wire wound around the inductor. (*This is a transformer*). Hook this coupling coil in series with a 50 ohm resistor (so as not to load down your oscillator output amplifier) and apply a small fraction of a volt from your oscillator at a frequency near where you calculate the resonance of your LC circuit will be for an intermediate setting of the variable capacitor (say 250pF).

With your scope switched to high input impedance, measure the voltage across your LC circuit with a X10 scope probe as you “tune” your variable capacitor. You need to use the X10 probe so you don’t create extra loss by effectively placing a resistance across the circuit. Observe the resonance. Do the reverse too: set your oscillator to a frequency near your resonance, and then change the oscillator frequency measuring the peak-to-peak voltage as you go, mapping out this resonance. Some of the elective experiments use this kind of circuit. Plot this curve and fit it with an appropriate function (you will do this in some of the elective experiments).

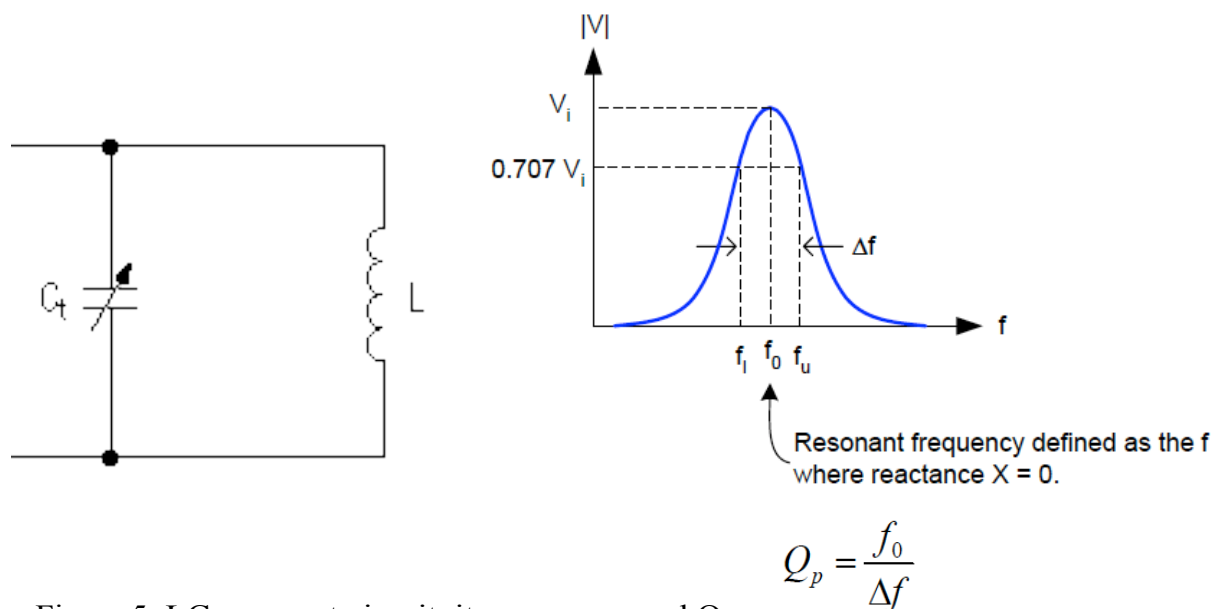


Figure 5: LC resonant circuit, its response, and Q.

There is an exact analogy between the RLC circuit and a mechanical harmonic oscillator:

$$m \frac{d^2x}{dt^2} + B \frac{dx}{dt} + kx = 0 \quad \text{damped harmonic oscillator}$$

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0 \quad \text{undriven RLC circuit}$$

where $x \sim q$ (charge), $L \sim m$, $k \sim 1/C$, and the damping coefficient $B \sim R$.

The sharpness of this resonance makes the LC circuit ideal for situations where you want to pass only signals in a very narrow range of frequencies (band pass filter) or, if hooked up differently, if you want to reject signals in a narrow range of frequencies (band reject, or “notch” filter).

The measure of the sharpness of the resonance is the Q, defined in Fig. 5. The Q is the resonant frequency divided by the full width of the resonance curve at the half power points (0.707 voltage points). This follows from the definition of the Q as:

Q (quality factor) of a circuit: determines how well the RLC circuit stores energy
 $Q = 2\pi (\text{max energy stored})/(\text{energy lost}) \text{ per cycle}$

From this definition of Q and the expression for stored energy $CV^2/2$, write an expression for Q in terms of L, C, and a small effective series resistance R. Repeat but instead for a large effective parallel resistance R.

How high a Q can you get with your circuit? Try 2-3 different values of your variable capacitor over its whole range, measuring the Q at each of the corresponding resonant frequencies. We'll have a contest! The winner gets their name entered into the PHY Advanced Lab Hall of Fame. What limits the Q of your particular setup? Observe what happens if you switch your probe to X1.

The above definition of Q suggests a complementary way of measuring it – in the time domain. Remember the time domain- frequency domain duality. To measure the Q in the time domain measure how long it takes to ring down by some factor, or equivalently measure the decrease of voltage in one cycle. So you need to shock excite your LC circuit, just as you would strike a tuning fork.

Set up a pulse generator giving a few volts pulse height at a pulse repetition frequency of 1-10 KHz and ~50-100ns pulse widths [explain why so narrow], and feed this into your coupling coil+50ohm resistor. Look at the response by triggering your scope via a BNC Tee on the pulse generator output connected to CH2, measuring the voltage waveform on CH1 with your X10 probe of course. After adjusting trigger and channel gains, you may see something like Fig. 6. Tune the variable capacitor and see the Q change by observing the change in waveform decay. Derive the Q from this time-domain technique, for your best Q setting from the previous frequency domain measurement. What limits your measurement precision in this second method? You might consider measuring the voltage fractional decay over N cycles.

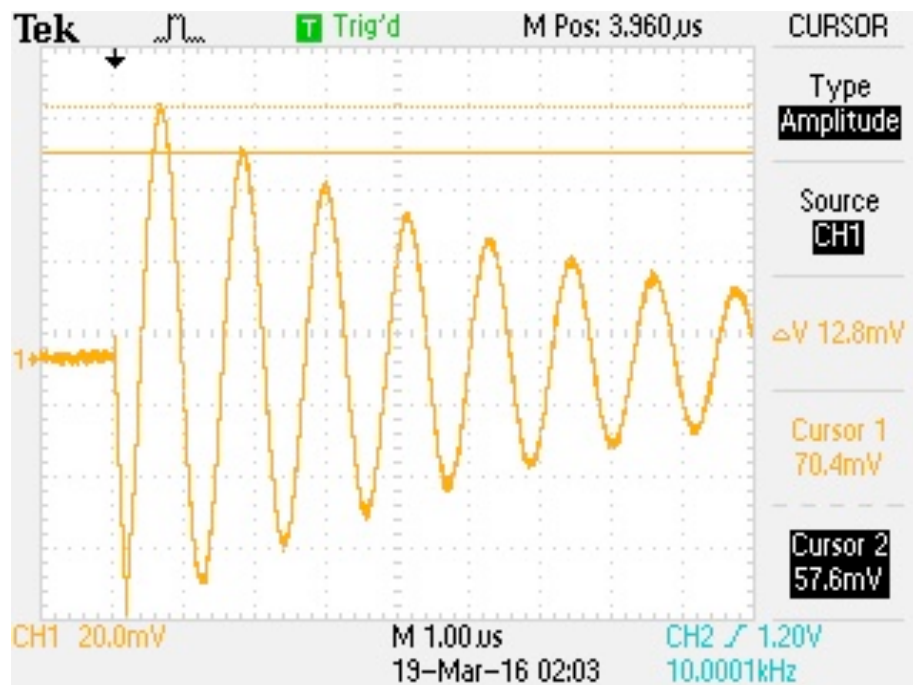


Figure 6: Shock excited oscillator: your LC circuit response to a pulse. Cursors were used in the amplitude mode to measure the decay of voltage in one cycle.

